

Dirac or Inverse Seesaw Neutrino Masses with $B - L$ Gauge Symmetry and S_3 Flavour Symmetry

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Abstract

Many studies have been made on extensions of the standard model with $B - L$ gauge symmetry. The addition of three singlet (right-handed) neutrinos renders it anomaly-free. It has always been assumed that the spontaneous breaking of $B - L$ is accomplished by a singlet scalar field carrying two units of $B - L$ charge. This results in a very natural implementation of the Majorana seesaw mechanism for neutrinos. However, there exists in fact another simple anomaly-free solution which allows Dirac or inverse seesaw neutrino masses. We show for the first time these new possibilities and discuss an application to neutrino mixing with S_3 flavour symmetry.

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I. INTRODUCTION

The standard model (SM) of quarks and leptons is free of gauge anomalies. If it is extended to include an extra $U(1)$ gauge group, the conditions for the absence of gauge anomalies impose nontrivial constraints. Nevertheless, interesting discoveries of such possibilities have been made. One example [1] adds a lepton triplet per family to the SM. Another [2] adds a number of new superfields to the minimal supersymmetric standard model (MSSM) of three families. On the other hand, the most studied extension is that of $U(1)_{B-L}$. Using the particle content of the standard model, all triangle gauge anomalies are zero except for

$$\sum U(1)_{B-L}^3 = -3 \quad (1)$$

This is easily solved with the addition of three (right-handed) singlet neutrinos, which contribute $-3(-1)^3 = +3$. Note that the gauge-gravitational anomaly is also zero because $-3(-1) = +3$. Numerous studies have been made regarding this model of $U(1)_{B-L}$.

In this paper, we point out that there is another simple choice for three families [3–5]. Let $\nu_{Ri} \sim n_i$ under $B - L$, then $n_{1,2,3} = (+5, -4, -4)$ yields

$$-(+5)^3 - (-4)^3 - (-4)^3 = +3 \quad (2)$$

as well. In fact, since $-(5) - (-4) - (-4) = +3$, the mixed gauge-gravitational anomaly is also zero. Now the standard-model Higgs doublet $(\phi^+, \phi^0)^T$ does not connect ν_L with ν_R and there is no neutrino mass. Consider then three heavy Dirac singlet fermions $N_{L,R}$, all transforming as -1 under $B - L$. They do not change the anomaly conditions, but now in the case of ν_{R2} and ν_{R3} , $(\bar{\nu}_L, \bar{N}_L)$ is linked to (ν_R, N_R) through the 2×2 mass matrix as follows

$$M_{\nu,N} = \begin{pmatrix} 0 & m_0 \\ m_3 & M \end{pmatrix} \quad (3)$$

where m_0 comes from $\langle \phi^0 \rangle$ and m_3 comes from $\langle \chi_3 \rangle$, assuming the Yukawa coupling $\bar{N}_L \nu_R \chi_3$, i.e. $\chi_3 \sim +3$ under $B - L$. Now, the invariant mass M is naturally large, so the Dirac

seesaw [6] yields a small neutrino mass $m_3 m_0 / M$. In the conventional $U(1)_{B-L}$ model, $\chi_2 \sim +2$ under $B-L$, is chosen to break the gauge symmetry, so that ν_R gets a Majorana mass and lepton number L is broken to $(-1)^L$. Here, $\chi_3 \sim +3$ means that L remains a conserved global symmetry, with $\nu_{L,R}$ and $N_{L,R}$ all having $L = 1$. Since $\nu_{R1} \sim +5$ does not connect with ν_L or N_L directly, there is one massless neutrino in this case. However, the dimension-five operator $\bar{N}_L \nu_{R3} \chi_3^* \chi_3^* / \Lambda$ is allowed by $U(1)_{B-L}$ and would give it a very small Dirac mass. Alternatively, a second scalar $\chi_6 \sim 6$ may be added.

The above assignment of $B-L = +3$ for the scalar field χ_3 ; is the new idea of this paper, which was not recognized anywhere before. Because the pairing of two neutral fermions of the same chirality in this model always results in a nonzero $B-L$ charge not divisible by 3 units, it is impossible to construct an operator of any dimension for a Majorana mass term which violates $B-L$. Hence the statement that $B-L$ is reduced to a conserved global $U(1)$ symmetry is definitely robust.

Another possible outcome of the above scenario is obtained with the choice of two complex scalar fields $\chi_2 \sim 2$ and $\chi_6 \sim 6$ under $U(1)_{B-L}$. In this case, ν_L is not connected to $\nu_{R1,R2} \sim -4$. It is connected however to $N_{L,R}$ through the mass matrix spanning $(\bar{\nu}_L, N_R, \bar{N}_L)$ as follows:

$$M_{\nu,N} = \begin{pmatrix} 0 & m_0 & 0 \\ m_0 & m'_2 & M \\ 0 & M & m_2 \end{pmatrix} \quad (4)$$

where m_2 and m'_2 come from χ_2 . This is then an inverse seesaw [7–9], i.e. $m_\nu \simeq m_0^2 m_2 / M^2$. In the case of $\nu_{R3} \sim +5$, the corresponding mass matrix spanning $(\bar{\nu}_L, N_R, \bar{N}_L, \nu_{R3})$ is given by

$$M_{\nu,N} = \begin{pmatrix} 0 & m_0 & 0 & 0 \\ m_0 & m'_2 & M & 0 \\ 0 & M & m_2 & m_6 \\ 0 & 0 & m_6 & 0 \end{pmatrix} \quad (5)$$

where m_6 comes from χ_6 . Thus ν_{R3} also gets an inverse seesaw mass $\simeq m_6^2 m'_2 / M^2$. Note

that Eq. (5) is the 4×4 analog of the 3×3 lopsided seesaw discussed in [10].

In this scheme, lepton number L is broken to $(-1)^L$ for ν_L and ν_{R3} , and $N_{1,2,3}$ are heavy pseudo-Dirac fermions. However, $\nu_{R1,R2}$ remain massless and may only be produced in pairs. They are thus good dark-matter candidates if they become massive. This may be accomplished with a third scalar $\chi_8 \sim 8$.

II. THE S_3 FLAVOUR SYMMETRY: $\mu - \tau$ SECTOR

In this section we expand upon the first scenario discussed in the previous section, using two singlet scalar fields transforming as $+3$ under the $B - L$ symmetry, in analogy to having ν_{R2} and ν_{R3} transforming as -4 . This allows us to use the doublet representations of the non-Abelian discrete symmetry group S_3 to understand the leptonic family structure as indicated by present neutrino oscillation data and other experimental constraints. Our application of S_3 will be different from those of Ref. [4, 5] since we have the three $N_{L,R}$ fields in addition to $\nu_{R1,R2,R3}$. We will first work out the details of our model for the simpler case of $\mu - \tau$ sector and show how one can use S_3 symmetry to obtain maximal mixing. Then we will discuss the implications of imposing S_3 symmetry to the full 3×3 mixing matrix. However, before presenting our model let us briefly discuss the S_3 symmetry group.

The S_3 group is the smallest non-Abelian discrete symmetry group and is the group of the permutation of three objects. It consists of six elements and is also isomorphic to the symmetry group of the equilateral triangle. It admits three irreducible representations 1, $1'$ and 2 with the tensor product rules

$$1 \times 1' = 1', \quad 1' \times 1' = 1, \quad 2 \times 1 = 2, \quad 2 \times 1' = 2, \quad 2 \times 2 = 1 + 1' + 2 \quad (6)$$

In this work, following the earlier works [11, 12], we choose to work with the complex representation of the S_3 group. In the complex representation, if

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \in \mathbf{2} \quad \Rightarrow \quad \begin{pmatrix} \phi_2^\dagger \\ \phi_1^\dagger \end{pmatrix}, \quad \begin{pmatrix} \psi_2^\dagger \\ \psi_1^\dagger \end{pmatrix} \in \mathbf{2} \quad (7)$$

such that

$$\begin{aligned}
\phi_1\psi_2 + \phi_2\psi_1, \quad \phi_2^\dagger\psi_2 + \phi_1^\dagger\psi_1 &\in \mathbf{1} \\
\phi_1\psi_2 - \phi_2\psi_1, \quad \phi_2^\dagger\psi_2 - \phi_1^\dagger\psi_1 &\in \mathbf{1}' \\
\begin{pmatrix} \phi_2\psi_2 \\ \phi_1\psi_1 \end{pmatrix}, \quad \begin{pmatrix} \phi_1^\dagger\psi_2 \\ \phi_2^\dagger\psi_1 \end{pmatrix} &\in \mathbf{2}
\end{aligned} \tag{8}$$

With this brief summary of S_3 group and its irreducible representations, we now move on to constructing an S_3 invariant $\mu - \tau$ sector. In later section we will generalize the discussion of this section to the full 3×3 mixing matrix.

A. The S_3 invariant $\mu - \tau$ sector

We denote the left handed lepton doublets by $L^\alpha = (\nu_L^\alpha, l_L^\alpha)^\text{T}$ where $\alpha = \mu, \tau$; the right handed charged leptons are denoted as μ_R, τ_R and the right handed neutrinos as ν_R^μ, ν_R^τ . Also, let us denote the heavy singlet fermions as $N_{L,R}^i$; $i = 2, 3$. The ‘‘Standard Model like’’ scalar doublet is denoted by $\Phi = (\phi^+, \phi^0)^\text{T}$ and the singlet scalars are denoted by $\chi_{2,3}$.

Let the $B - L$ charge and S_3 assignment of the above fields be as shown in Table I.

Fields	$B - L$	S_3	Fields	$B - L$	S_3
L^μ	-1	$1'$	L^τ	-1	1
μ_R	-1	$1'$	τ_R	-1	1
N_L^2	-1	$1'$	N_L^3	-1	1
N_R^2	-1	$1'$	N_R^3	-1	1
Φ	0	1			
$\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix}$	-4	2	$\begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix}$	3	2

TABLE I. The $B - L$ and S_3 charge assignment for the fields.

The S_3 and $B - L$ invariant Yukawa $\mathcal{L}_Y^{(2)}$ interaction can be written as

$$\mathcal{L}_Y^{(2)} = \mathcal{L}_{L^\alpha l_R}^{(2)} + \mathcal{L}_{L^\alpha N_R}^{(2)} + \mathcal{L}_{N_L N_R}^{(2)} + \mathcal{L}_{N_L \nu_R}^{(2)} \quad (9)$$

where

$$\begin{aligned} \mathcal{L}_{L^\alpha l_R}^{(2)} &= y_\mu \bar{L}^\mu \Phi \mu_R + y_\tau \bar{L}^\tau \Phi \tau_R \\ \mathcal{L}_{L^\alpha N_R}^{(2)} &= g_2 \bar{L}^\mu \hat{\Phi}^* N_R^2 + g_3 \bar{L}^\tau \hat{\Phi}^* N_R^3 \\ \mathcal{L}_{N_L N_R}^{(2)} &= M_2 \bar{N}_L^2 N_R^2 + M_3 \bar{N}_L^3 N_R^3 \\ \mathcal{L}_{N_L \nu_R}^{(2)} &= f_2 \bar{N}_L^2 \otimes \left[\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix} \otimes \begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \right]_{1'} + f_3 \bar{N}_L^3 \otimes \left[\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix} \otimes \begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \right]_1 \end{aligned} \quad (10)$$

In writing (10), we used the notation $\hat{\Phi}^* = i\tau_2 \Phi^* = (\phi^0, \phi^-)^T$. Moreover, y_μ, y_τ are the Yukawa couplings of the charged lepton sector whereas f_i, g_i and M_i denote the dimensionless coupling constants between the leptons and the heavy fermions.

After symmetry breaking the scalar fields get vacuum expectation values (VEVs) $\langle \phi^0 \rangle = v$, $\langle \chi_i \rangle = u_i$; $i = 2, 3$. Then the mass matrix relevant to charged leptons is given by

$$\mathcal{M}_l = v \begin{pmatrix} y_\mu & 0 \\ 0 & y_\tau \end{pmatrix} \quad (11)$$

Also, the 4×4 mass matrix spanning $(\bar{\nu}_L^\mu, \bar{\nu}_L^\tau, \bar{N}_L^2, \bar{N}_L^3)$ and $(\nu_R^\mu, \nu_R^\tau, N_R^2, N_R^3)^T$ of neutrinos and the heavy fermions is given by

$$\mathcal{M}_{\nu, N} = \begin{pmatrix} 0 & 0 & g_2 v^* & 0 \\ 0 & 0 & 0 & g_3 v^* \\ f_2 u_3 & -f_2 u_2 & M_2 & 0 \\ f_3 u_3 & f_3 u_2 & 0 & M_3 \end{pmatrix} \quad (12)$$

As remarked earlier, the mass terms M_i between the heavy fermions can be naturally large, so we can block diagonalize (12) assuming that $f_i, g_i \ll M_i$. The block diagonalized mass matrix of light neutrinos is given by

$$\begin{aligned}
\mathcal{M}_\nu &= m_{N_L \nu_R}^{(2)} \frac{1}{M_{N_L N_R}^{(2)}} m_{L^\alpha N_R}^{(2)} \\
&= v^* \begin{pmatrix} \frac{f_2 g_2}{M_2} u_3 & -\frac{f_2 g_3}{M_3} u_2 \\ \frac{f_3 g_2}{M_2} u_3 & \frac{f_3 g_3}{M_3} u_2 \end{pmatrix}
\end{aligned} \tag{13}$$

where $m_{L^\alpha N_R}^{(2)}$, $M_{N_L N_R}^{(2)}$ and $m_{N_L \nu_R}^{(2)}$ are the 2×2 mass matrices obtained respectively from $\mathcal{L}_{L^\alpha N_R}^{(2)}$, $\mathcal{L}_{N_L N_R}^{(2)}$ and $\mathcal{L}_{N_L \nu_R}^{(2)}$ terms of (10). This light neutrino mass matrix can be further diagonalized by the bi-unitary transformation. The neutrino masses and the mixing angles obtained from (13) will be dependent on the specific values of the coupling constants f_i, g_i, M_i as well as the VEVs v, u_i ; $i = 2, 3$. For sake of illustration we explicitly compute them for two simplifying scenario leading to maximal mixing.

Case I: $f_2 = f_3 = f$, $g_2 = g_3 = g$, $M_2 = M_3 = M$, $u_2 \neq u_3$.

In this case the neutrino mass matrix becomes

$$\mathcal{M}_\nu^I = \frac{f g v^*}{M} \begin{pmatrix} u_3 & -u_2 \\ u_3 & u_2 \end{pmatrix} \tag{14}$$

Case II: $f_2 = f_3 = f$, $u_2 = u_3 = u$, $\frac{g_2}{M_2} \neq \frac{g_3}{M_3}$.

In this case the neutrino mass matrix becomes

$$\mathcal{M}_\nu^{II} = f u v^* \begin{pmatrix} \frac{g_2}{M_2} & -\frac{g_3}{M_3} \\ \frac{g_2}{M_2} & \frac{g_3}{M_3} \end{pmatrix} \tag{15}$$

Both the mass matrices in (14) and (15) can be written as

$$\mathcal{M}_\nu = \kappa \begin{pmatrix} a & -b \\ a & b \end{pmatrix} \tag{16}$$

where

$$\kappa = \begin{cases} \frac{f g v^*}{M} \\ f u v^* \end{cases}, \quad a = \begin{cases} u_3 \\ \frac{g_2}{M_2} \end{cases}, \quad b = \begin{cases} u_2 & \text{In Case I} \\ \frac{g_3}{M_3} & \text{In Case II} \end{cases} \tag{17}$$

The mass matrix in (16) can be easily diagonalized by a bi-unitary transformation leading to the neutrino masses $\nu_2 = \sqrt{2}\kappa a$ and $\nu_3 = \sqrt{2}\kappa b$ with mixing angle $\theta_{23} = \frac{\pi}{4}$ where κ, a, b for both cases are given in (17). Therefore, S_3 allows us to understand maximal mixing in the $\mu - \tau$ sector.

III. THE COMPLETE S_3 INVARIANT LEPTON SECTOR

The $B - L$ charge and S_3 assignment of the fields for the full lepton sector is as shown in Table II.

Fields	$B - L$	S_3	Fields	$B - L$	S_3
L^e	-1	$1'$	e_R	-1	$1'$
L^μ	-1	$1'$	μ_R	-1	$1'$
L^τ	-1	1	τ_R	-1	1
N_L^1	-1	$1'$	N_R^1	-1	$1'$
N_L^2	-1	$1'$	N_R^2	-1	$1'$
N_L^3	-1	1	N_R^3	-1	1
Φ	0	1	ν_R^e	5	$1'$
$\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix}$	-4	2	$\begin{pmatrix} \chi^2 \\ \chi^3 \end{pmatrix}$	3	2

TABLE II. The $B - L$ and S_3 charge assignment for the fields.

The S_3 and $B - L$ invariant Yukawa \mathcal{L}_Y interaction can be written as

$$\mathcal{L}_Y = \mathcal{L}_{L^\alpha l_R} + \mathcal{L}_{L^\alpha N_R} + \mathcal{L}_{N_L N_R} + \mathcal{L}_{N_L \nu_R} \quad (18)$$

where

$$\begin{aligned}
\mathcal{L}_{L^\alpha l_R} &= y'_e \bar{L}^e \Phi e_R + y'_{12} \bar{L}^e \Phi \mu_R + y'_{21} \bar{L}^\mu \Phi e_R + y'_\mu \bar{L}^\mu \Phi \mu_R + y_\tau \bar{L}^\tau \Phi \tau_R \\
\mathcal{L}_{L^\alpha N_R} &= g'_{11} \bar{L}^e \hat{\Phi}^* N_R^1 + g'_{12} \bar{L}^e \hat{\Phi}^* N_R^2 + g'_{21} \bar{L}^\mu \hat{\Phi}^* N_R^1 + g'_{22} \bar{L}^\mu \hat{\Phi}^* N_R^2 + g_{33} \bar{L}^\tau \hat{\Phi}^* N_R^3 \\
\mathcal{L}_{N_L N_R} &= M'_{11} \bar{N}_L^1 N_R^1 + M'_{12} \bar{N}_L^1 N_R^2 + M'_{21} \bar{N}_L^2 N_R^1 + M'_{22} \bar{N}_L^2 N_R^2 + M_{33} \bar{N}_L^3 N_R^3 \\
\mathcal{L}_{N_L \nu_R} &= \frac{f'_{11}}{\Lambda} (\bar{N}_L^1 \nu_R^e) \otimes \left[\begin{pmatrix} \chi_3^* \\ \chi_2^* \end{pmatrix} \otimes \begin{pmatrix} \chi_3^* \\ \chi_2^* \end{pmatrix} \right]_1 + \frac{f'_{21}}{\Lambda} (\bar{N}_L^2 \nu_R^e) \otimes \left[\begin{pmatrix} \chi_3^* \\ \chi_2^* \end{pmatrix} \otimes \begin{pmatrix} \chi_3^* \\ \chi_2^* \end{pmatrix} \right]_1 \\
&\quad + f'_{12} \bar{N}_L^1 \otimes \left[\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix} \otimes \begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \right]_{1'} + f'_{22} \bar{N}_L^2 \otimes \left[\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix} \otimes \begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \right]_{1'} \\
&\quad + f_{33} \bar{N}_L^3 \otimes \left[\begin{pmatrix} \nu_R^\mu \\ \nu_R^\tau \end{pmatrix} \otimes \begin{pmatrix} \chi_2 \\ \chi_3 \end{pmatrix} \right]_1
\end{aligned} \tag{19}$$

As before, in writing (19), we used the notation $\hat{\Phi}^* = i\tau_2 \Phi^* = (\phi^0, \phi^-)^T$. Moreover, y_α are the Yukawa couplings of the charged leptons whereas f_{ij} , g_{ij} and M_{ij} denote the dimensionless coupling constants between the leptons and the heavy fermions.

At this point we like to remark that in (19) there is still a freedom to redefine a few fields (i.e. the pairs $N_L^1 - N_L^2$, $N_R^1 - N_R^2$ and $e_R - \mu_R$) in a way that certain couplings can be made equal to zero. For sake of later convenience we choose to use this freedom of field redefinition to make $f'_{11} = M'_{12} = y'_{21} = 0$. Moreover, we relabel the remaining non-zero couplings of these redefined fields as $f'_{ij} \rightarrow f_{ij}$, $g'_{ij} \rightarrow g_{ij}$, $M'_{ij} \rightarrow M_{ij}$.

Now, after symmetry breaking the scalar fields get VEVs $\langle \phi^0 \rangle = v$, $\langle \chi_i \rangle = u_i$; $i = 2, 3$. Then the mass matrices relevant to charged lepton is given by

$$\mathcal{M}_l = v \begin{pmatrix} y_e & y_{12} & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \tag{20}$$

The mass matrix (20) can be readily diagonalized by bi-unitary transformation. In the limit of $y_e \ll y_\mu$ we get

$$\begin{aligned}\theta_{12}^l &\approx \tan^{-1} \left(\frac{-y_{12}}{y_\mu} \right); & m_e &\approx v y_e \cos \theta_{12}^l \\ m_\mu &\approx v (y_\mu \cos \theta_{12}^l - y_{12} \sin \theta_{12}^l); & m_\tau &\approx v y_\tau\end{aligned}\quad (21)$$

If $y_{12} = y_\mu$ then maximal mixing is achieved i.e. $\theta_{12}^l = -\frac{\pi}{4}$, with $m_\mu = \sqrt{2} v y_\mu$.

Also, the 6×6 mass matrix spanning $(\bar{\nu}_L^e, \bar{\nu}_L^\mu, \bar{\nu}_L^\tau, \bar{N}_L^1, \bar{N}_L^2, \bar{N}_L^3)$ and $(\nu_R^e, \nu_R^\mu, \nu_R^\tau, N_R^1, N_R^2, N_R^3)^T$ of neutrinos and the heavy fermions is given by

$$\mathcal{M}_{\nu, N} = \begin{pmatrix} 0 & 0 & 0 & g_{11} v^* & g_{12} v^* & 0 \\ 0 & 0 & 0 & g_{21} v^* & g_{22} v^* & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{33} v^* \\ 0 & f_{12} u_3 & -f_{12} u_2 & M_{11} & 0 & 0 \\ \frac{f_{21}}{\Lambda} u_2^* u_3^* & f_{22} u_3 & -f_{22} u_2 & M_{21} & M_{22} & 0 \\ 0 & f_{33} u_3 & f_{33} u_2 & 0 & 0 & M_3 \end{pmatrix} \quad (22)$$

As remarked earlier, the mass terms M_{ij} between the heavy fermions can be naturally large, so we can block diagonalize (22) assuming that $f_{ij}, g_{ij} \ll M_{ij}$. The block diagonalized mass matrix of light neutrinos is given by

$$\begin{aligned}\mathcal{M}_\nu &= m_{N_L \nu_R} \frac{1}{M_{N_L N_R}} m_{L^\alpha N_R} \\ &= v^* \begin{pmatrix} \frac{(g_{21} M_{11} - g_{11} M_{21}) f_{12} u_2}{M_{11} M_{22}} & \frac{(g_{22} M_{11} - g_{12} M_{21}) f_{12} u_2}{M_{11} M_{22}} & \frac{-f_{12} g_{33} u_3}{M_{33}} \\ \frac{(g_{21} M_{11} - g_{11} M_{21}) f_{22} u_2 + f_{21} g_{11} M_{22} u_6}{M_{11} M_{22}} & \frac{(g_{22} M_{11} - g_{12} M_{21}) f_{22} u_2 + f_{21} g_{12} M_{22} u_6}{M_{11} M_{22}} & \frac{-f_{22} g_{33} u_3}{M_{33}} \\ \frac{(g_{21} M_{11} - g_{11} M_{21}) f_{33} u_2}{M_{11} M_{22}} & \frac{(g_{22} M_{11} - g_{12} M_{21}) f_{33} u_2}{M_{11} M_{22}} & \frac{f_{33} g_{33} u_3}{M_{33}} \end{pmatrix} \end{aligned} \quad (23)$$

where we have written $u_6 = \frac{u_2^* u_3^*}{\Lambda}$. Also, the 3×3 mass matrices $m_{L^\alpha N_R}$, $M_{N_L N_R}$ and $m_{N_L \nu_R}$ are obtained from the terms $\mathcal{L}_{L^\alpha N_R}$, $\mathcal{L}_{N_L N_R}$ and $\mathcal{L}_{N_L \nu_R}$ of (19), respectively. This light neutrino mass matrix can be further diagonalized by the bi-unitary transformation. The neutrino masses and the mixing angles obtained from (23) will be dependent on the specific values of the coupling constants f_{ij}, g_{ij}, M_{ij} as well as the VEVs $v, u_i; i = 2, 3$.

In the simplifying case of $g_{ij} = g$ and $M_{ij} = M$ we get

$$\mathcal{M}_\nu = \frac{gv^*}{M} \begin{pmatrix} 0 & 0 & -f_{12}u_3 \\ f_{21}u_6 & f_{21}u_6 & -f_{22}u_3 \\ 0 & 0 & f_{33}u_3 \end{pmatrix} \quad (24)$$

Diagonalizing the mass matrix (24) we have

$$\begin{aligned} \theta_{12}^\nu &\approx 0; & \theta_{13}^\nu &\approx \tan^{-1} \left(\frac{f_{12}}{f_{33}} \right); & \theta_{23}^\nu &\approx \tan^{-1} \left(\frac{f_{22}}{\sqrt{f_{12}^2 + f_{33}^2}} \right) \\ m_1^\nu &\approx 0; & m_2^\nu &\approx \frac{\sqrt{2(f_{12}^2 + f_{33}^2)}f_{21}g|v|}{M\sqrt{f_{12}^2 + f_{22}^2 + f_{33}^2}}|u_6|; & m_3^\nu &\approx \frac{\sqrt{f_{12}^2 + f_{22}^2 + f_{33}^2}g|v|}{M}|u_3| \end{aligned} \quad (25)$$

Since, $u_6 \ll u_3$, we have a normal hierarchy pattern with two nearly massless neutrinos and one relatively heavy neutrino. Moreover, the massless neutrino will also gain small mass, if any of the M_{ij} 's or g_{ij} 's are not equal to M or g respectively. Also, if they deviate significantly from these values then one can possibly recover degenerate or inverted hierarchy patterns also.

Now, if U_l and U_ν are the mixing matrices of the charged leptons and neutrinos respectively, then the PMNS mixing matrix is given by

$$U_{\text{PMNS}} = U_l^\dagger U_\nu \quad (26)$$

Taking $y_{12} = y_\mu$, $f_{12} = -\frac{f_{33}}{2}$ and $f_{22} = \sqrt{f_{12}^2 + f_{33}^2}$ in (21) and (25) we get $\theta_{23}^\nu = -\theta_{12}^l = \frac{\pi}{4}$ and $\theta_{13}^\nu = \tan^{-1}(-\frac{1}{2})$ which gives PMNS mixing angles consistent¹ with present $3 - \sigma$ limits of global fits obtained from experiments [13].

¹ With $\theta_{23}^\nu = -\theta_{12}^l = \frac{\pi}{4}$ and $\theta_{13}^\nu = \tan^{-1}(-\frac{1}{2})$ we get the value of mixing angles as $\theta_{12} = 30.29^\circ$, $\theta_{13} = 7.53^\circ$ and $\theta_{23} = 50.36^\circ$. The values of θ_{12} and θ_{13} are slightly below the lower limits (30.59° , 7.62° respectively) quoted in [13]. A much better fit can be obtained if we take slightly different values, for instance taking $\theta_{12}^l = -50^\circ$, $\theta_{13}^\nu = -29.39^\circ$ and $\theta_{23}^\nu = \frac{\pi}{4}$ gives us $\theta_{12} = 33.26^\circ$, $\theta_{13} = 9.00^\circ$ and $\theta_{23} = 51.41^\circ$.

The Quark Sector

In our minimal model with only one doublet scalar, the quark sector can be accommodated in a simple way if both the left handed quark doublets $Q_L^i = (u_L^i, d_L^i)^T$, $i = 1, 2, 3$ and the right handed quark singlets u_R^i, d_R^i ; $i = 1, 2, 3$ transform as 1 of S_3 .

A better understanding of the quark sector can be obtained if, to our minimal model, we add more doublet scalars transforming non-trivially under S_3 . One such example for quark sector, albeit in context of a different model for lepton sector, has already been worked out in [11, 12]. We plan to present a similar extension of our minimal model in a future work.

IV. CONCLUSIONS

The idea that $B - L$ should be a gauge symmetry has been around for a long time. The minimal version of three ν_R transforming as -1 is very well-known. Other exotic variants are possible with extra particles, such as the two models recently proposed [14, 15]. Here we point out the simple anomaly-free solution of three ν_R 's transforming as $+5, -4, -4$. We show how these assignments may be used to obtain seesaw Dirac neutrino masses, as well as inverse seesaw Majorana neutrino masses. We then apply S_3 symmetry to the first case, and obtain realistic neutrino and charged-lepton mass matrices with a mixing pattern consistent with experiments.

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